boundary-element method (BEM) is based on BIE. However, they thought that the BIE was a common numerical technique already established and confirmed generally in this field, so that they did not refer to it individually.

In the original paper, the authors aimed to emphasize the facility of the application of the BEM, which is an "element method" and whose discretizing technique is like that of the finite-element method (FEM). These facts cause the BEM to become a very powerful numerical method. It is very easy to perform programming for computers. In addition, it adopts simple and general expressions (for example, the equation having a general variable-a single scalar potential), so that the formulation is performed about the scalar Helmholtz's equation, and when actual problems are treated, a proper boundary condition is imposed on the above potential. Moreover, the same program can be used for different cases (for example, for the case of sound problems). Its govering equation is also the scalor Helmholtz's equation, but its boundary condition is different from that of the electromagnetic field problem.

Finally, the authors would like to thank Dr. N. Morita for his remarks and for providing [10], [20], and [21].

REFERENCES

[1] K. K. Mei and J. Van Baldel, "Low-frequency scattering by rectangular cylinders," *IEEE Trans. Antennas Propagat.*, vol. AP-11, pp. 52–56, Jan 1963.
[2] K. K. Mei and J. Van Bladel, "Scattering by perfectly-conducting rectangular cylinders," *IEEE Trans. Antennas Propagat.*, vol. AP-11, pp. 185–192, Mar. 1963.
[3] R. F. Harrington, *Field Computation by Moment Methods*. New York: MacMillan, 1968.
[4] R. Mittra, Ed., *Computer Techniques for Electromagnetics* Oxford: Pergamon, 1973.
[5] R. Mittra, Ed., *Numerical and Asymptotic Techniques in Electromagnetics* (Topics in Applied Physics, Vol. 3). Berlin: Springer, 1975.
[6] D. S. Jones, "Numerical methods for antenna problems," *Proc Inst. Elec. Eng.*, vol. 121, pp. 573–582, July 1974.
[7] B E. Spielman and R. F. Harrington, "Waveguides of arbitrary cross section by solution of a nonlinear integral eigenvalue equation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 578–585, Sept. 1972.
[8] N Okamoto, "Computer-aided design of *H*-plane waveguide junctions with full-height ferrites of arbitrary shape," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 315–321, Apr. 1979.
[9] V V Solodukhov and E N. Vasil'ev, "Diffraction of a plane electromagnetic wave by a dielectric cylinder of arbitrary cross section," *Sov. Phys.-Tech. Phys.*, vol. 15, pp. 32–36, July 1970.
[10] N. Morita, "Analysis of scattering by a dielectric rectangular cylinder by means of integral equation formulation," *Trans. Inst. Elec. Commun. Eng Japan*, vol 57-B, pp. 72–80, Oct. 1974.
[11] T.-K. Wu and L. L. Tsai, "Numerical analysis of electromagnetic fields in biological tissues," *Proc. IEEE*, vol. 62, pp. 1167–1168, Aug. 1974.
[12] N. Okamoto, "A method for problems of scattering by gyrotropic composite cylinders," *Trans. Inst. Elec. Commun. Eng. Japan*, vol. 58-B, pp. 269–276, June 1975.
[13] J R. Mautz and R. F. Harrington, "Electromagnetic scattering from a homogeneous material body of revolution," *Arch. Elek. Übertragung.*, vol. 33, pp. 71–80, Feb. 1979.
[14] T-K. Wu and L. L. Tsai, "Scattering from arbitrarily-shaped lossy dielectric bodies of revolution," *Radio Sci.*, vol. 12, pp. 709–718, Sept./Oct. 1977.
[15] K. A. Zaki and A. R. Neureuther, "Scattering from a perfectly conducting surface with a sinusoidal height profile," *IEEE Trans. Antennas Propagat.*, vol. AP-19, pp. 208–214, Mar. 1971
[16] K. K. Mei, "Unimoment method of solving antenna and scattering problems," *IEEE Trans. Antennas Propagat.*, vol. AP-22, pp. 760–766, Nov. 1974.
[17] H. A. Schenck, "Improved integral formulation for acoustic radiation problems," *J. Acoust. Soc. Am.*, vol. 44, pp. 41–58, July 1968
[18] J -C. Bolomey and W. Tabbara, "Numerical aspects on coupling between complementary boundary value problems," *IEEE Trans Antennas Propagat.*, vol. AP-21, pp. 356–363, May 1973.
[19] C. A. Klein and R Mittra, "An application of the 'Condition Number' concept to the solution of scattering problems in the presence of the interior resonant frequencies," *IEEE Trans. Antennas Propagat.*, vol. AP-23, pp. 431–435, May 1975.
[20] N. Morita, "Surface integral representations for electromagnetic scattering from dielectric cylinders," *IEEE Trans. Antennas Propagat.*, vol. AP-26, pp. 261–266, Mar. 1978.
[21] N. Morita, "Resonant solutions involved in the integral equation approach to scattering from conducting and dielectric cylinders," *IEEE Trans. Antennas Propagat.*, vol. AP-27, pp. 869–871, Nov. 1979.
[22] J. R Mautz and R. F. Harrington, "*H*-field, *E*-field, and combined field solutions for bodies of revolution," *Arch. Elek Übertragung.*, vol. 32, pp. 157–164, Apr. 1978.

Comments on "Limitations of the Cubical Block Model of Man in Calculating SAR Distributions"

MARK J. HAGMANN, MEMBER, IEEE

The above paper[1] raised some serious questions regarding the accuracy of three-dimensional block model solutions obtained using a pulse-function basis. While I am in qualitative agreement with about half of the numerical results presented in the paper, I most strongly disagree with most of the interpretations which the authors have made using those results. It is my belief that it is possible to obtain high accuracy with block model solutions if sufficient care is used in their implementation. I have chosen to use a pulse-function basis with block models of man since this appears to allow the model to have much greater detail than is possible with more elaborate bases.

The paper incorrectly stated that I have given "an upper limit on the dimensions of cells for the required accuracy" and inferred that such a limit was satisfied in their solutions. In earlier work with one of the authors (Durney), it was shown that the size of each cell must not be much greater than the reciprocal of the magnitude of the complex propagation vector, but this was presented as a condition that is necessary but not sufficient for convergence [2]. Pulse functions are only appropriate if the electric field is slowly varying over the volume of each cell. The electric field will have sizable variation within some objects even in static solutions. One case in point is the dielectric cube which the authors unfortunately chose to use as an example.

The solution for a 27-cell block model of a dielectric cube, as presented in the article, is very far from convergence. While an exact solution is not available for the dielectric cube, it is generally known that the electric field is highly heterogeneous near the corners and edges. While I have not obtained a solution for a cube having the exact parameters used by the authors, the results of earlier studies [3], [4], as well as recent work using as many as 2744 cells, suggests that the fields near corners and edges are sufficiently intense that the true average SAR would be several times greater than that calculated for a 27-cell block model. I am not surprised that subdividing the cell at the center of the cube had little effect since it is well known that at low frequencies the electric field at the center of a cube is the same as that at the center of a sphere, and the solution for a small number of cells is more like that for a sphere than a cube. I am also not surprised that subdividing a cell at a corner or edge of the cube caused a significant change in the SAR since these are regions where the 27-cell solution has the greatest error.

Manuscript received August 27, 1984.
The author is with the National Institutes of Health, Bethesda, MD 20205.
[1] H. Massoudi, C. H. Durney, and M F. Iskander, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp 746–752, Aug 1984.

0018-9480/85/0400-0347$01.00 ©1985 IEEE

Two different formulations have been used with block models of man. One was developed by Chen [5] and the other was developed by Hagmann [6], who made high-frequency corrections of a routine written earlier by Hohmann for use in geophysical prospecting [7]. The second procedure will be referred to as the high-frequency Hohmann (HFH). Both use a pulse-function basis with point matching but they differ in the methods used for approximation of the integrals required for evaluation of the matrix elements. Hohmann transformed the charge portion of the integrals to surface integrals. The resulting expressions are integrable without requiring principal-value corrections. Chen did not make this transformation and thus was forced to use principal-value corrections in evaluation of the singular self terms. The approximations made in evaluation of the terms for cell-to-cell coupling were much less severe in the HFH formulation than those used by Chen.

The extended boundary condition method (EBCM) has recently been modified so that accurate solutions may be obtained for a prolate spheroidal model of man at frequencies above resonance [8]. I have chosen to use this model as a standard for testing block model solutions since it is the most man-like model for which an exact solution is known and, therefore, falls most directly within my charge at the National Institutes of Health. One of the authors of [1] (Iskander), who was also one of the developers of the extension of the EBCM, has provided me with the average SAR for a prolate spheroidal model of man at 100 and 225 MHz with E (vertical) polarization. Tests of the HFH formulation using block models with as many as 3048 cells gave values of average SAR within about 6 percent of those obtained using the modified EBCM. Comparisons using local SAR values as well as other polarizations await the supply of requested additional EBCM solutions.

When the Chen formulation was used with the prolate spheroidal model of man, the errors in average SAR were significantly greater than those obtained using the HFH formulation. These increased errors were found to be caused by inaccuracies in the matrix elements. Subsequently, numerical quadrature was used to assure accurate evaluation of the matrix elements, and there was good agreement with the HFH formulation. In [1], it is stated that the Chen formulation has been tested using a block model of a lossy dielectric sphere. In those tests, the computed values of the local electric-field intensity were in good agreement with the exact solution only for the case of extremely low dielectric contrast ($\epsilon_r = 1.016 - j0.2809$) [9]. Tests of the Chen formulation using dielectric properties more appropriate for biological objects were not successful [10].

Massoudi *et al.* [1] stated that solutions obtained using a pulse-function basis cannot satisfy boundary conditions at the cell surfaces. They implied that such problems are more serious for inhomogeneous models. The use of a pulse-function basis is certainly an approximation, and the degree to which the boundary conditions are approximated is different for various solutions. In the block model solutions obtained using the HFH formulation with large numbers of cells for a prolate spheroidal model of man, there was typically only about 2-percent variation in the electric-field intensity between adjacent cells. Such a small cell-to-cell variation lends confidence in the use of the pulse-function basis and also constitutes a reasonably good approximation of boundary conditions. In the case of inhomogeneous models, there is an apparent problem if the electric-field intensity is approximately the same in two adjacent cells, and if the cells have different dielectric properties, then there must be a jump in the value of the electric flux density normal to the boundary. This problem is resolved when it is recognized that the matrix elements, which are obtained from the electric-field integral equation (EFIE), allow not only for a constant polarization current density within each cell but also for a possible charge density at each cell surface. The surface integral of the charge term in the HFH formulation represents the effects of that surface charge density. It is easily shown that this charge density is the value required to satisfy the jump in electric flux density if the cells have different dielectric properties. The point is that boundary conditions are built into the EFIE, and the accuracy of their approximation increases as one approaches a converged solution for either homogeneous or inhomogeneous block models.

It is essential that the array of cubes used in discretization be a best-fit of the object to be modeled. Increasing the number of cells in a model by subdividing the existing cells and retaining the same outer boundary may fail to increase the accuracy of a block model solution. This is particularly true if the subdivision emphasizes corners and edges not present in the object being modeled. In the tests made for comparison with the modified EBCM, much care was taken to obtain a best-fit of the prolate spheroid in each discretization. A "bad" 2368-cell model was made for comparison by keeping the outer boundary of a 296-cell model and dividing each cube into 8. The average SAR obtained using the HFH formulation with the "bad" model at a frequency of 100 MHz had an error that was greater than that for the 296-cell model and about twice that for "good" models having numbers of cells comparable with 2368.

It is easily shown that the average SAR calculated using one cell as a block model is appropriate for a sphere at low frequencies and not for a cube. Similarly, the solution for a column of single cells corresponds to a circular cylinder rather than a rectangular solid. This is because many cells would be required in order to allow for the variation of the electric-field intensity near corners and edges. It is for this reason that I infer that the solution for a block model of man is more representative of man than it would be of a figure having the corners and edges which are apparent in the model. It is not surprising that Massoudi *et al.* [1] found that the local SAR changed when cells at corners and edges of their block model of man were subdivided. They were simply working toward the solution for a model having corners and edges and away from that for man. Similar calculations have been made by others [11], [12]. I would have anticipated that the values obtained by this subdivision in [1] would have been much greater than those which were reported, and I suspect that this difference is due to their use of the Chen formulation.

Reply[2] *by Habib Massoudi, Carl H. Durney, and Magdy F. Iskander*[3]

If we interpret Dr. Hagmann's comments correctly, the substance of his remarks (along with our reply) is as follows.

1) He believes that it is possible to obtain high accuracy with block model solutions if sufficient care is used, and pulse-basis functions appear to allow much greater detail in a model than more elaborate basis functions.

Our reply: We believe that pulse-basis functions, because of their simplicity, are the obvious choice wherever they can provide satisfactory results. However, we have not been able to obtain satisfactory accuracy in calculating internal electric-field distribu-

[2] Manuscript received November 2, 1984.
[3] The authors are with the Department of Electrical Engineering, University of Utah, Salt Lake City, UT 84112

tions using these types of basis functions. If there is a way to obtain suitable accuracy, we would certainly like to see data that demonstrates it. In other words, none of the results that were published by Dr. Hagmann or others who used the pulse-basis function have demonstrated acceptable accuracy in calculating the electric-field distributions within the man model.

2) Dr. Hagmann claims that we incorrectly stated that he has given "an upper limit on the dimensions of cells for the required accuracy," and we inferred that such a limit was satisfied in our solutions.

Our reply: The sentence in [1] to which Dr. Hagmann refers is vague, since "these authors" could be taken to refer to any or all of the authors of the three papers to which we referred [2], [5], [9] in the previous two sentences. Our point in [1] is that the calculated internal field values were not sufficiently accurate even though the cells were smaller than upper limits stated by several authors [2], [5], [9]. Specifically, we made our mathematical cells much smaller than $\lambda_{0/10}$ (λ_0 is the free-space wavelength) and also smaller than the reciprocal of the magnitude of the complex propagation vector which is suggested by Hagmann *et al.* [2]. We said nothing about how accurate and mathematically valid these criteria are. To obtain the best possible results from the cubical block model using pulse-basis functions, we simply tried to follow the guidelines regarding the cell size which were published by Hagmann *et al.* [2] and Livesay and Chen [5].

3) According to Dr. Hagmann, our choice of the dielectric cube as an example was unfortunate because pulse functions work only when the fields are slowly varying in each cell, which is not true for the 27-cell model that we chose. Subdividing a corner cell is expected to cause significant error in the SAR because that is where the 27-cell solution has the greatest error. The solution for a block model of man is more representative of an actual man than it would be of a figure having the corners and edges that are apparent in the model.

Our reply: We chose the dielectric cube because it is a good example for showing why pulse functions are not adequate in many cases for calculating internal electric-field distributions. As we pointed out in [1], the rapid spatial variation of the fields near corners and, in general, at dielectric discontinuities appears to be the main reason why pulse-basis functions are inadequate. Furthermore, the data in [1] for block models of standing and sitting humans show that the same problems that exist in the dielectric cube also exist in typical block models of humans, which was where we first encountered the problem. These results indicate that the SAR distribution data calculated using pulse-basis functions in 114-cell [1] and 180-cell [6] block models may not be very accurate. Dr. Hagmann's claim that "the solution for a block model of man is more representative of an actual man than it would be of a figure having the corners and edges which are apparent in the model" does not seem to help justify the inaccurate field distributions obtained when the more rounded dielectric sphere was modeled. If the choice of the dielectric cube was unfortunate, how can one justify the inaccurate results for the spherical model in [9] and [10] of Dr. Hagmann's comments? The problem involves more than just modeling, as will be clarified in the remaining part of our comments.

4) He believes that the HFH method is better than the Chen formation because it includes a transformation of volume integrals to surface integrals, which do not require principal-value corrections in the evaluation of the singular self terms. Calculation of *average* SAR in prolate spheroids using the HFH method with as many as 3048 cells gave values within about 6 percent of those obtained from the IEBCM.

Our reply: Dr. Hagmann's comparison of his recent results with the *average* SAR values obtained using the IEBCM is irrelevant to the point we were trying to make regarding the inaccurate field *distributions* obtained in the cubical block models. As indicated in the title of our paper [1], we are basically concerned with the limitations of the cubical block model of man in calculating SAR *distributions*. If the HFH method was used in calculating SAR's in the 180-cell block model at 10 MHz [5], [6], it does not appear to avoid the problems discussed above, since the ratio of SAR's in adjacent cells is as high as eight to one. In work that we have completed [13] since [1] was published, we have found that the free-space Green's function integral equation (FGIE), stated as (1) in [1], gives more accurate values of internal electric-field distribution in models of dielectric spheres using pulse-basis functions than the dyadic Green's function integral equation (DGIE) used in both the Chen formation and the Hagmann formulation [4]. Although we have not proved this, we believe that there are two reasons for the better results. First, the FGIE is less singular than the DGIE, and second, the FGIE contains a term that explicitly corresponds to the source surface charge density, while the DGIE does not, although it certainly includes the effect of this surface change density implicitly. Our data lead us to believe that the accuracy of internal field calculations depends significantly on the description of the surface charge density. On the basis of our understanding and experience, we do not believe that the HFH will provide satisfactory accuracy for internal field distributions in a 180-cell model of man. The fact that the average SAR calculations in prolate spheroidal models is accurate within about 6 percent does not indicate that the SAR distributions will be accurate enough, since the data in [1] indicate that the local SAR values can change significantly while the average SAR changes but little

5) Dr. Hagmann points out that even though pulse functions cannot satisfy the boundary conditions at the cell walls exactly, they can satisfy them approximately, especially if there is little variation in the electric-field intensity between adjacent cells. In inhomogeneous models, there must be a jump in the electric flux density normal to the boundary between two cells of different permittivity, which is an apparent problem that is resolved when it is recognized that the boundary conditions are built into the electric-field integral equation.

Our reply: We agree that the boundary conditions are built into the electric-field integral equation, and the boundary conditions are satisfied in some approximate way by pulse functions. However, if adjacent cells in an inhomogeneous model have different permittivities, the boundary conditions could require the pulse functions on one face of the cube to be one value, and on another face of the cube to be quite another value. Since a pulse function has only one value in a cell, the boundary conditions at both surfaces could not be satisfied well at all, even in an approximate sense. Even when a large number of cells is used, the boundary conditions at interfaces between adjacent cells of different permittivities would not be satisfied very well by pulse functions. We believe that the boundary conditions are very important to the accuracy of the solution because of the surface charge density induced at a discontinuity in permittivity. As we mentioned above, our results lead us to believe that the accuracy of the numerical solution of the integral equation depends strongly on the adequacy with which the surface charge density is accounted for in the numerical solution. If this is true, then the limitation of the pulse functions in satisfying boundary conditions, and therefore in describing surface charge density at permittivity discontinuities, may be the main reason for numerical

inaccuracies. We would like to emphasize that some groups have already realized these basic limitations of the pulse-basis functions and particularly their inappropriateness in satisfying the boundary conditions between cells. It is these limitations that lead these research groups to utilize more sophisticated basis functions such as the roof-top function [14] and the linear basis functions [13]. We believe that these functions allow better description of the fields within each cell and help to satisfy the boundary conditions between cells. This, together with better modeling of the geometry of the object (e.g., by replacing the cubical cells by polyhedral cells, as we recommended in [1]), should significantly improve the SAR *distributions* calculated by the method of moments.

In summary, we believe that the data presented in [1] indicate that there are serious questions about the convergence of solutions obtained from the use of pulse-basis functions in the moment-method solution of the electric-field integral equation for the internal field distribution. The reasons for these deficiencies have not been rigorously proven, but we have proposed some explanations based on our understanding and experience. We believe that it is generally accepted by those working in numerical electromagnetics that the use of pulse-basis functions can give satisfactory results for the average SAR, but not for the internal field distribution. Whether or not satisfactory results for internal field distribution can be obtained by using pulse functions and a very large number of cubical cells remains to be demonstrated, but in most numerical calculations, there is a point at which the accuracy begins to decrease as the size of the cells is made smaller.

Each of the various numerical electromagnetic techniques in common use has its advantages and disadvantages, and must be used with care for any given application to ensure that it provides useful results. The use of pulse-basis functions in cubical cells in the moment-method solution is no exception, and it may turn out that when the results are compared in terms of matrix size, the use of more complicated basis functions, such as linear basis functions, may provide better accuracy in calculating internal field distributions. However, it does turn out, it seems clear, that reasonably accurate calculation of internal field distributions by the moment method will be expensive because either the use of a large number of cells or the use of more complex basis functions than pulse functions will be required, both of which entail very large matrices.

References

[1] H Massoudi, C. H Durney, and M F Iskander, "Limitations of the cubical block model of man in calculating SAR distributions," *IEEE Trans. Microwave Theory Tech*, vol MTT-32, pp 746–752, Aug. 1984.

[2] M J. Hagmann, O. P Gandhi, and C H Durney, "Upper bound on cell size for moment-method solutions," *IEEE Trans Microwave Theory Tech*, vol MTT-25, pp 831–832, Oct. 1977.

[3] M. J Hagmann, O P Gandhi, and C H Durney, "Improvement of convergence in moment-method solutions by the use of interpolants," *IEEE Trans. Microwave Theory Tech*, vol. MTT-26, pp. 904–908, Nov 1978

[4] M. J Hagmann, "Numerical studies of absorption of electromagnetic energy by man," Ph D. dissertation, Dep Elec. Eng, Univ. of Utah, Salt Lake City, UT, Dec 1978

[5] D E Livesay and K M Chen, "Electromagnetic fields induced inside arbitrarily shaped biological bodies," *IEEE Trans Microwave Theory Tech*, vol MTT-22, pp 1273–1280, Dec. 1974

[6] M J. Hagmann, O P Gandhi, and C H Durney, "Numerical calculation of electromagnetic energy deposition for a realistic model of man," *IEEE Trans. Microwave Theory Tech*, vol MTT-27, pp 804–809, Sept 1979

[7] G W Hohmann, "Three-dimensional induced polarization and electromagnetic modeling," *Geophysics*, vol 40, pp 309–324, Apr 1975

[8] M. F. Iskander, A. Lakhtakia, and C. H. Durney, "A new procedure for improving the solution stability and extending the frequency range of the EBCM," *IEEE Trans. Antennas Propagat.*, vol AP-31, pp. 317–324, Mar. 1983

[9] J. W Penn and D. K. Cohoon, "Analysis of a Fortran program for computing electric field distributions in heterogeneous penetrable nonmagnetic bodies of arbitrary shape through application of tensor Green's functions," USAF School of Aerospace Medicine, Brooks Air Force Base, TX, SAM-TR-78-40, Dec. 1978.

[10] D K. Cohoon, Dep Math, Temple University, Philadelphia, PA, personal communication

[11] J F. Deford, O P. Gandhi, and M. J. Hagmann, "Moment-method solutions and SAR calculations for inhomogeneous models of man with large number of cells, "*IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp 848–851, Oct 1983.

[12] E. H Wissler, "Final Report. Thermal and nonthermal effects of microwave radiation," National Institutes of Health grant GM27844, Dep Chem Eng, Univ. Texas at Austin, Mar 1983

[13] C. T Tsai, H. Massoudi, C. H. Durney, and M F Iskander, "Improved calculations of SAR distributions in biological models," presented at the Fifth Annual Conf Bioelectromagnetics Society.

[14] D H Schaubert, D R Wilton, and A W. Glisson, "A tetrahedral modeling method for electromagnetic scattering by arbitrarily shaped inhomogenerous dielectric bodies," *IEEE Trans. Antennas Propagat.*, vol. AP-32, pp. 77–85, Jan. 1984

Correction to "Interaction of the Near-Zone Fields of a Slot on a Conducting Sphere with a Spherical Model of Man"

SHI-GUO ZHU AND K M. CHEN

In the above paper,[1] we have assumed that on the surface of the conducting sphere, the electric field exists only on the slot aperture and zero field elsewhere. This assumption is unrealistic. For a realistic conducting sphere, the slot field can excite a normal component of the electric field on the spherical surface, implying the existence of induced surface charge. With this modified assumption, (6) of the above paper should be modified as follows:

$$\begin{aligned}\vec{E}'^{(0)}(a,\theta,\phi) &= \sum_{n=0}^{\infty}\sum_{m=-n}^{n}\left[A'^{(0)}_{mn}\vec{M}^{h}_{mn}(a,\theta,\phi)\right.\\ &\quad\left.+B'^{(0)}_{mn}\vec{N}^{h}_{mn}(a,\theta,\phi)\right]\\ &= \hat{\theta}\frac{1}{a}\delta(\theta-\theta_0)f(\phi)\\ &\quad+\hat{r}\sum_{n=0}^{\infty}\sum_{m=-n}^{n}B^{r(0)}_{mn}\left[\vec{N}^{h}_{mn}(a,\theta,\phi)\right]_r \qquad (6)\end{aligned}$$

where

$$f(\phi)=\begin{cases}\cos(\pi\phi/2\alpha), & \text{for } -\alpha\leqslant\phi\leqslant\alpha\\ 0, & \text{elsewhere.}\end{cases}$$

As a result, the coefficients $A^{r(0)}_{mn}$ and $B^{r(0)}_{mn}$ of (7) and (8) are

Manuscript received November 19, 1984.

S G Zhu is with Physics Department of Sichuan University, Chengdu, Sichuan, People's Republic of China

K M. Chen is with Department of Electrical Engineering and Systems Science, Michigan State University, E Lansing, MI 48824.

[1] Shi-Guo Zhu, K. M Chen, and H R Chuang, *IEEE Trans Microwave Theory Tech.*, vol. MTT-32, pp 784–795, Aug 1984.